UPPSC-AE 2025

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Electrical Engineering

Elements of Electrical Machines

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY PUBLICATIONS

Electrical Machines

Contents

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CHAPTER 1 Electromagnetic System and General Concepts of Rotating Machines

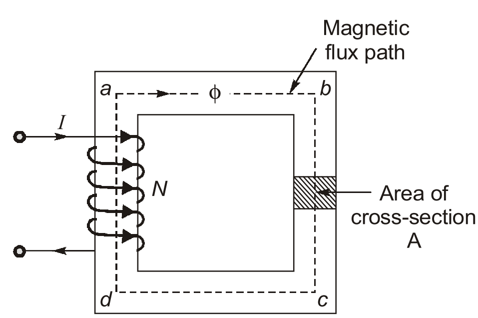
1.1 Introduction

The electromagnetic system is an essential element of all rotating electric machinery and electromechanical device and static devices like the transformer. Electromechanical energy conversion takes place via the medium of a magnetic field or electrical field, but most practical converters use magnetic field as the coupling medium between electrical and mechanical systems.

This chapter deals with the general concepts of mmf, emf and torque in rotating electrical machines. The basic emf and torque expressions are applicable to both DC and AC machines as the fundamental principles underlying their operation are same, but final forms of expressions differ only due to difference in constructional features.

1.2 Magnetic Circuits and Related Terminology

- **Magnetic circuits:** It is a complete closed path containing lines of magnetic flux which is usually generated either by permanent magnet or electromagnets.



(Magnetic circuit with N-turns and current I)

Magnetomotive Force (MMF)

- In a magnetic circuit, the magnetic flux is due to the presence of a magnetomotive force same as in an electric circuit, the current is due to the presence of a electromotive force.
- The mmf is created by a current flowing through one or more turns.

MMF = Current × Number of turns in the coil

i.e., I = MMF = NI (ampere-turns) or (ATs)

Reluctance (R_l)

Opposition offered by the magnetic circuit to magnetic flux is called **reluctance**.

$$R_l = \frac{l}{\mu A} \text{ AT/Wb}$$

where, l = length of the magnetic path
A = area of cross-section normal to flux path, m^2
μ = $\mu_0 \cdot \mu_r$ = permeability of the magnetic material
μ_r = relative permeability of the magnetic material
μ_0 = permeability of free space = $4\pi \times 10^{-7}$ H/m.

Magnetic Flux (ϕ)

- The magnetic flux may be defined as the magnetomotive force per unit reluctance.

$$\phi = \frac{\text{MMF}}{\text{Reluctance}} \quad \text{or} \quad \phi = \frac{N.I.\,\mu_o \mu_r \cdot A}{l}\ \text{Wb}$$

- Direction of magnetic flux produced by coil can be found by right-hand grip rule.

Permeance (P)

Reciprocal of reluctance is called permeance.

$$\text{Permeance} = \frac{1}{\text{Reluctance}} = \frac{\mu_o\, \mu_r\, A}{l}\ \text{Wb/AT}$$

Magnetic Field Density (B)

It is defined as the magnetic flux per unit cross-sectional area of the core.

$$B = \frac{\text{Magnetic flux, }\phi}{\text{core area, A}} = \frac{NI \cdot \mu_o\, \mu_r}{l}\ \text{Wb/m}^2$$

Magnetic Field Intensity (H)

The magnetomotive force per unit length of magnetic circuit is termed as the magnetic field intensity.

$$H = \frac{\text{MMF}}{\text{mean length of magnetic circuits}} \quad \text{or} \quad H = \frac{NI}{l}\ \text{ATs/m}$$

NOTE

- In magnetic system there are no magnetic insulators. Even in the best known magnetic insulator air, the flux can be established.
- Energy is needed for establishing the required flux once the requisite flux is created then no more energy is needed in maintaining it.

Example - 1.1 A magnetic circuit has 180 turns coil and cross sectional area 5×10^{-4} m^2 and length of magnetic circuit is 25 cm. The value of relative permeability for current 2A and flux of 0.35 mWb is:

(a) 401.88 (b) 1215.27
(c) 386.83 (d) None

Solution : (c)

Given,

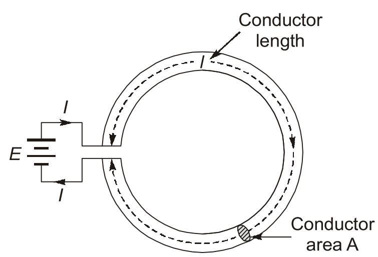
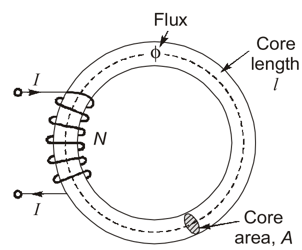
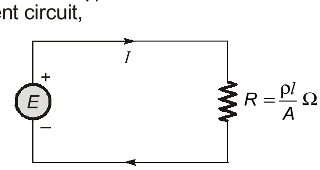
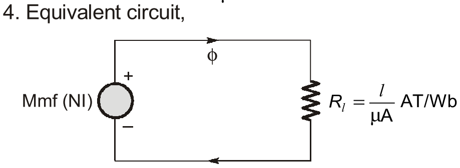
$N = 180$, $A = 5 \times 10^{-4}$ m^2, $l = 25$ cm $= 25 \times 10^{-2}$ m, $I = 2$A, $\phi = 0.35 \times 10^{-3}$ Wb

$$\therefore \quad \phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{NI\mu_0 \cdot \mu_r \cdot A}{l}$$

$$\Rightarrow \quad 0.35 \times 10^{-3} = \frac{180 \times 2 \times 4\pi \times 10^{-7} \times \mu_r \times 5 \times 10^{-4}}{25 \times 10^{-2}}$$

$$\Rightarrow \quad \mu_r = 386.83$$

1.3 Comparison Between Magnetic and Electric Circuits

Comparison between Magnetic and Electric Circuits	
Electrical Circuit	**Magnetic Circuit**
	
A torodial copper ring of length *l*, cross-sectional area A is connected to emf *E* so that current *I* flows.	A torodial iron ring of length *l*, cross area A is excited by a coil of *N* turns carrying *I* amperes so that flux ϕ is produced.
Similarities:	**Similarities:**
1. Closed path for electrical current is called an electric circuit.	1. Closed path for the magnetic flux is called a magnetic circuit.
2. Driving force is emf *E*, volts	2. Driving force is Mmf = NI ATs
3. Resistance, $R = \frac{\rho \cdot l}{A}$ V/A or Ω	3. Reluctance, $R_l = \frac{l}{\mu A}$ AT/Wb
4. Equivalent circuit, 	4. Equivalent circuit, 
5. Circuit, $I = \frac{\text{Driving force}}{\text{Resistance}} = \frac{E}{R} A$	5. Magnetic flux, $\phi = \frac{\text{Driving force}}{\text{Reluctance}} = \frac{\text{Mmf}}{R_l}$ Wb
6. Current density, $J = \frac{I}{A}$ A/m^2	6. Magnetic flux density, $B = \frac{\phi}{A}$ T (or Wb/m^2)
7. Electric field intensity, $\varepsilon = \frac{E}{l}$ V/m Also, $\varepsilon = \frac{E}{l} = \frac{IR}{l} = \frac{I}{l} \cdot \frac{\rho l}{A} = \rho \cdot \frac{I}{A} = \rho \cdot J$ V/m	7. Magnetic field intensity, $H = \frac{\text{NI}}{l}$ AT/m Also, $H = \frac{\text{NI}}{l} = \frac{\text{Mmf}}{l} = \frac{\phi \cdot R_l}{l} = \frac{\phi}{l} \cdot \frac{l}{\mu A} = \frac{1}{\mu} \cdot \frac{\phi}{A} = \frac{1}{\mu} \cdot B$ AT/m
8. Conductivity of current path, $\sigma = 1/\rho$ So that, $J = \sigma \varepsilon$ A/m^2	8. Conductivity of current path, μ So that, $B = \mu H$ Wb/m^2 or T
Dissimilarities:	**Dissimilarities:**
1. The electrical current actually flows in an electric circuit. For the existence of this current, energy is drawn from the source continuously. This energy gets dissipated in resistance in the form of heat. 2. Electrical insulator confine the current to well defined paths.	1. Strictly speaking, magnetic flux does not flow. Energy is needed for establishing the required flux. Once the requisite flux is created, no more energy is needed in maintaining it. 2. There are no magnetic insulators. Even in the best known magnetic insulator air, the flux can be established.

Example - 1.2 Find the strength of magnetic field in a conductor 0.5 m long moving with velocity of 10 m/s and inducing an emf of 25 V and also magnetic field, velocity and length of conductor are mutually perpendicular to each other

(a) 0 T (b) 5 T
(c) 2 T (d) 10 T

Solution: (b)

For B, l, v to be perpendicular to each other,

induced emf is given as $e = Blv$

$\therefore$ $25 = B \times 0.5 \times 10$

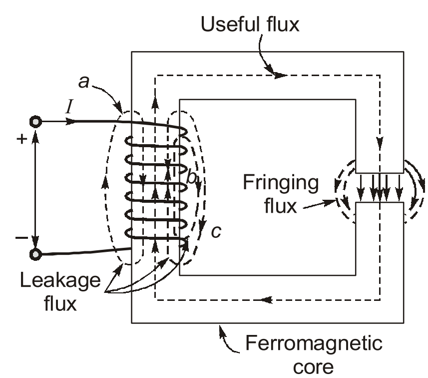
$\Rightarrow$ $B = 5\text{ T}$

1.4 Concept of Leakage Flux and Fringing

Leakage Flux

In Ideal magnetic circuits, all the flux produced by an exciting coil is confined to the desired magnetic path of negligible reluctance. But in practical magnetic circuits, a small amount of flux does follow a path through the surrounding air. Therefore, leakage flux may be defined as that flux which does not follow the intended path in a magnetic circuit.

Leakage flux does exist in all practical ferromagnetic device. Its effect on the analysis of electrical machinery is carried out by replacing it by an equivalent leakage reactance.



Fringing

At an air-gap in a magnetic core, the flux fringes out into neighboring air path as shown in the given. Longer the air gap, more is the flux fringing.

The effect of fringing flux is to increase the effective cross-sectional area of the air gap. As a result, flux density in the air gap is not uniform and average flux density gets reduced,

$$\therefore \quad B = \frac{\phi}{A}$$

If area of air gap increases then total area of core with consideration of air gap increases. Then average flux density gets reduced.

NOTE

- Longer the air gap, more is the flux fringing.
- Fringing flux reduces the average flux density.
- Leakage factor = $\dfrac{\text{Total flux handled by exciting winding}}{\text{Useful flux}}$

1.5 Induced emf

- Faraday's law of electromagnetic induction states that an e.m.f is induced in a coil when the magnetic flux linking this coil change with time.

- It is expressed as, $e = \frac{d\psi}{dt} = N\frac{d\phi}{dt}$

  where,
  e = e.m.f. induced in volts
  N = Number of turns in the coil
  Ψ = $N\phi$ = Flux linkages with the coil, Wb-turns
  t = time (seconds)

Lenz's Law

- When the coil circuit is closed, a current begins to flow in the coil. The direction of this induced emf, or induced current, is governed by Lenz's law. According to this law; the induced current develops a flux which always opposes the change responsible for inducing this current.

$$e = -\frac{d\psi}{dt} = -N\frac{d\phi}{dt}$$

- Lenz's law is based on the law: For every action, there is always an equal and opposite reaction. Thus in short, Lenz's law is effect opposes the cause.

Motional emf

- Flux cutting action occurs due to relative movement of coil and flux and the emf so induced is known as motional emf or speed emf.
- This principle is used in DC machines and synchronous machines.
- It is also called dynamically induced emf.

The e.m.f. induced 'e' by a conductor of length 'l' metres cutting a flux of density 'B' webers per square metre at a velocity 'v' metres per second is given by

$$e = B\,l\,v \text{ volts}$$

Provided that B, l and v are mutually perpendicular. If not, $B\,l\,v$ must be multiplied by the sine of the angle between any two of the three quantities, B, l and v.

Example - 1.3 The emf induced in a conductor of machine driven at 800 rpm for the parameters given below is

B_{peak} = 2 Wb/m^2, diameter of machine = 2.5 m, length of machine = 35 cm

(a) 64.85 V
(b) 73.33 V
(c) 68.51 V
(d) 90.5 V

Solution: (b)

$$\text{Area, } A = 2\pi r l = 2\pi \times \frac{D}{2} \times l = \pi D l$$

$$A = 3.14 \times 2.5 \times 0.35 = 2.75 \text{ m}^2$$

Flux, $\phi = B \cdot A = 2 \times 2.75 = 5.5$ Wb

induced emf, $e = \frac{N\phi}{T}$

$$= 5.5 \times \frac{800}{60} = 73.33 \text{ V}$$

1.6 Principle of Energy Conversion

- According to this principle, energy can neither be created nor destroyed, it can merely be converted from one form into another.
- In an energy conversion device, some energy is converted into the required form, some energy is stored and the rest is dissipated.
- Energy conversion process is basically a reversible process.
- Operating principles of energy conversion devices are similar, but their structural details differ depending upon their function.
- Coupling between the electrical and mechanical systems of devices is through magnetic or electric field.

Energy Balance Equation

- For a electrical motor:

  (Total electrical energy Input) = (Mechanical energy output) + (Total energy stored) + (Total energy dissipated)

  $W_{ei} = W_{mo} + (W_{es} + W_{ms})$ + (ohmic energy losses + coupling field energy losses) + (Energy losses in mechanical system)

  where, W_{ei} = Total electrical energy input from the supply mains.

  W_{mo} = The mechanical energy output

  $W_{es} + W_{em}$ = Energy stored in magnetic field + Energy stored in mechanical system

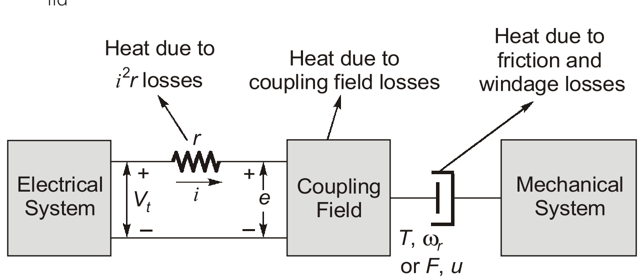
  = Total energy stored in any device and Total energy dissipated

  = Energy dissipated in electric circuit as ohmic loss + Energy dissipated as magnetic core loss (hysteresis and eddy-current losses) + Energy dissipated in mechanical system (Friction and windage losses etc.)

- For generator action:

  Total Mechanical energy input = (Electrical energy output) + (Total energy stored) + (Total energy dissipated)

- (W_{ei} – ohmic energy losses) = (W_{mo} + W_{ms} + Mechanical energy losses) + (W_{es} + Coupling field energy losses)
- $W_{elec} = W_{mech} + W_{fld}$



(*General representation of electromechanical energy conversion device*)

Example - 1.4 A physical system of electromechanical energy conversion, consists of a stationary part creating a magnetic field with electrical energy input and a moving part is kept fixed, the entire electrical energy input will be;

(a) stored in the magnetic field
(b) stored in the electric field
(c) divided equally between both the fields
(d) zero

Solution : (a)

∵ $W_{elec} = W_{mf} + W_{mech}$

where, W_{mech} = mechanical energy output

W_{elec} = electrical energy input

W_{mf} = energy stored in magnetic field

As movable part is fixed, hence $W_{mech} = 0$

Thus $W_{elec} = W_{mf}$

1.7 General Terms in Rotating Machines

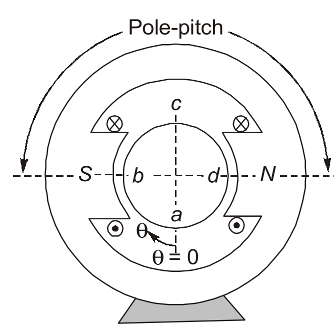


***Fig. (a) :** Elementary two pole machine*

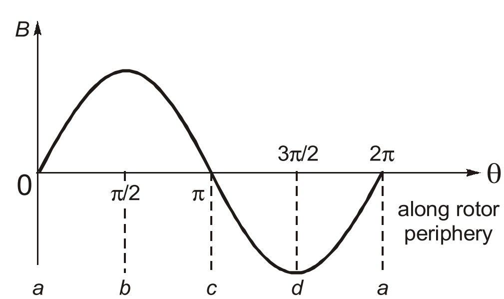


***Fig. (b) :** Flux density variation along air-gap periphery*

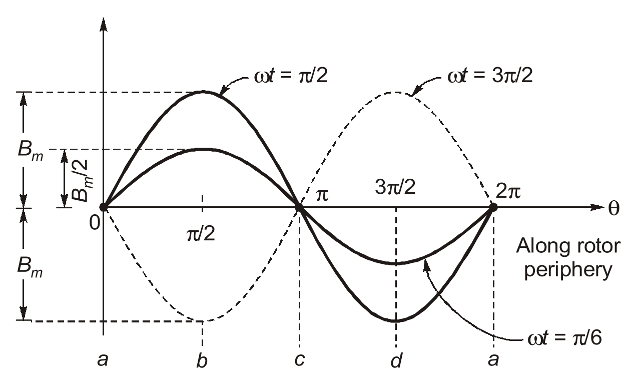


***Fig. (c) :** Pulsating flux*

Consider the figure (a), it shows an elementary two-pole machine, with its 2 field coils excited by direct current. The flux density at the point a in between the 2 poles will be zero. Under the centre of the pole indicated by point *b*, the flux density would be maximum positive; at c it is zero and at point '*d*' it is again maximum but negative.

South pole on the stator or north pole on the rotor, produces positive flux density.

1.7.1 Electrical And Mechanical Degree

- For a P-pole machine, $\frac{P}{2}$ cycles of emf will be generated in one revolution.

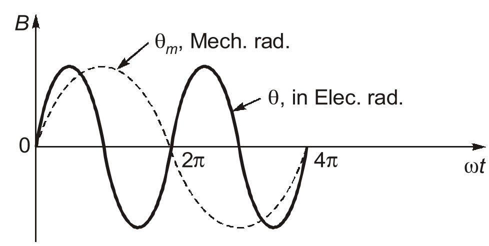
$$\theta_{elect} = \frac{P}{2}\theta_{mech}$$

or
$$\omega_e = \frac{P}{2}\omega_m$$

where ω_e is the angular speed in electrical radians per second and ω_m is the angular speed in mechanical radians per second.

- If the speed N is in rpm, then

$$f = \frac{PN}{120}\text{ Hz}$$



(*For* 4-*pole machine*)

Example - 1.5 If the torque angle of a 4-pole synchronous motor is 8 degree electrical, its value in mechanical angle is

(a) 4 degree
(b) 2 degree
(c) 0.5 degree
(d) 0.25 degree

[UPPSC]

Solution : (a)

Given,
$$P = 4$$
$$\theta_{elec} = 8 \text{ degree}$$

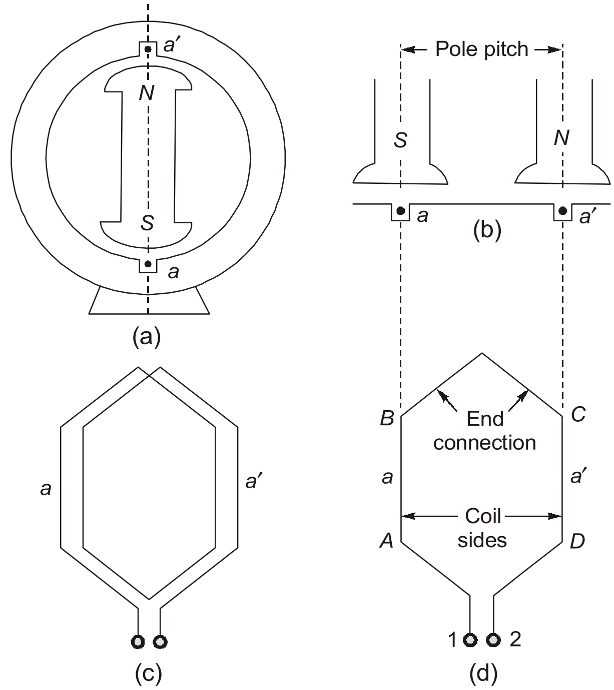
$\because$
$$\theta_{elec} = \frac{P}{2}\theta_{mech}$$

$\Rightarrow$
$$\theta_{mech} = \frac{2\theta_{elec}}{P} = \frac{2\times 8^\circ}{4} = 4^\circ$$

1.7.2 Coil Pitch and Pole Pitch

Coil

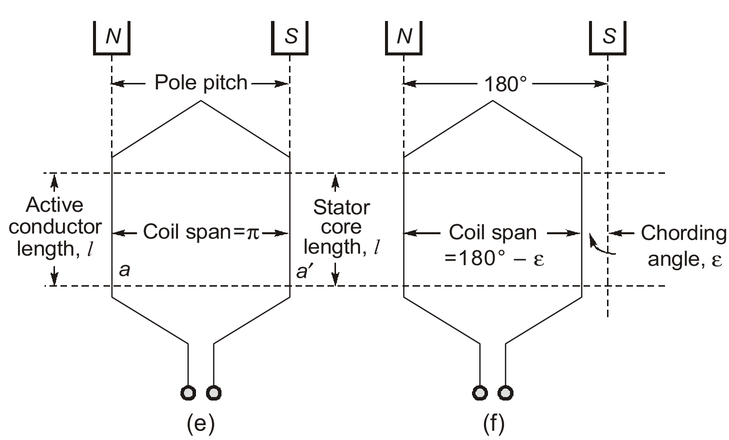
- Consider a 2 pole machine with one coil $a - a'$. The emf is generated in active lengths AB and CD only. These active lengths are called the two coil-sides of a coil.
- One turn consists of two conductors and one coil is made up of two coil sides.



Pole Pitch

- The peripheral distance between two adjacent poles is called pole pitch. It is always expressed in electrical degrees and pole pitch is always equal to 180 electrical degree or π-electrical radians.
- A coil with two coil sides 180° (electrical) space degree apart (or one pole pitch apart) is called a full pitch coil.

Coil Pitch

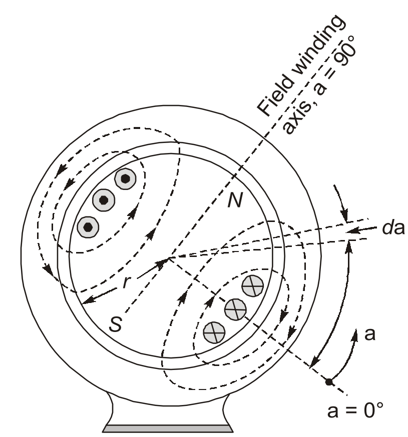


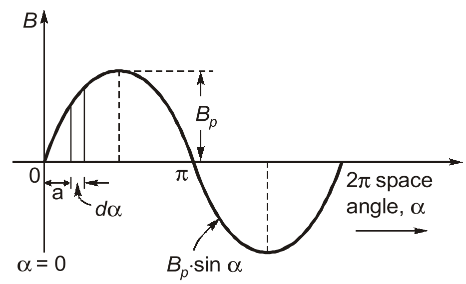
- Coil-span (or coil-pitch) is defined as the distance between the two coil-sides of one coil. Coil-span is measured in terms of electrical degrees, coil-sides or slots.
- Coil-span is less than 180°; this coil is, therefore, called a short-pitch, or chorded coil.
- Chording angle ($\in$) is defined as the angle by which coil-span departs from 180° electrical space degrees.

NOTE

- If chording angle ($\in$), then coil span = 180° – $\in$
- For full pole pitch, coil span = 180°

1.7.3 Flux Per Pole





Pertaining to flux per pole

- Consider figure shown above, where the field windings are taken on the rotor.
- When α = 0°, the flux density B is zero, when α = 0°, the flux density B is zero, when α = 90°, B is maximum say B_p, when α = 180°. B is again zero.
- The flux density B can be expressed as,

$$B = B_p \cdot \sin\alpha$$

- For a 2-pole machine, flux per pole = $2 \cdot B_p \cdot l \cdot r$

  where, l = axial length of armature core

  r = radius of armature core

- For a P-pole machine, flux per pole is given by $\frac{4}{P} B_p \cdot lr$
- Flux per pole (ϕ) can be written as

  ϕ = (Average value of constant-amplitude flux density wave under one pole) × (Area pertaining to one pole of the flux density wave)

$$= (B_{av})\left(\frac{2\pi r l}{P}\right)$$

$\because$ $$B_{av} = \frac{2}{\pi} B_p$$

$$\text{Total flux per pole} = \left(\frac{2}{\pi} B_p\right) \cdot \left(\frac{2\pi r l}{P}\right) = \frac{4}{P} B_p \cdot r l$$

Example - 1.6 In a 4 pole machine, what is the flux per pole produced, if the armature length is l and radius is r and B_p is peak value of sinusoidal flux?

(a) $4\, B_p \,.\, l \cdot r$

(b) $2\, B_p \,.\, l \cdot r$

(c) $B_p \,.\, l \cdot r$

(d) $\frac{B_p \cdot l \cdot r}{2}$

Student's Assignment

Q.1 Iron is removed from the iron cored coil so that the coil becomes air-cored coil. Inductance of this air-cored coil will
(a) increase
(b) decrease
(c) remain the same
(d) increase or decrease depending upon the coil configuration

Q.2 To eliminate the fifth harmonic a short-pitched coil should have a short-pitching angle of
(a) 36°
(b) 18°
(c) 15°
(d) 72°

[UPPSC}

Q.3 For a linear electromagnetic circuit, the following statement is true.
(a) Field energy is equal to the co-energy.
(b) Field energy is greater than the co-energy.
(c) Field energy is lesser than the co-energy.
(d) Co-energy is zero.

Q.4 The coupling field used between the electrical and mechanical systems in an energy conversion devices is:
(a) Magnetic field
(b) Electric field
(c) Either (a) or (b)
(d) None of these

Q.5 The part of coil in which EMF is generated is known as
(a) coil side
(b) coil span
(c) coil overhang
(d) end connections

Q.6 A circular metallic disc is placed in a vertical magnetic field of constant induction in the downward direction. If the disc is rotated in a horizontal plane, the mmf induced will be ______.
(a) zero
(b) constant and independent of disc size
(c) increasing radially in the outward direction
(d) decreasing radially in the outward direction

Q.7 The mutual inductance between two closely coupled coils is 1 H. Now the turns of one coil is decreased to half and those of the other is doubled. The new value of mutual inductance would be
(a) $\frac{1}{4}$ H
(b) 2 H
(c) $\frac{1}{2}$ H
(d) 1 H

Q.8 For electromechanical energy conversion, a magnetic field is employed as the medium rather than electric field because
(a) the stored energy density for practicable field strength is low in the electric field
(b) the electric field presents insulation problem
(c) the specific magnetic loss is more than the specific dielectric loss
(d) None of the above

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Q.9 Chording (short-pitching) of ac winding
(a) increases the harmonic content of the voltage wave but reduces winding copper.
(b) increases the harmonic content of the voltage wave and also increases winding copper.
(c) reduces the harmonic content of the voltage wave but increases winding copper.
(d) reduces the harmonic content of the voltage wave and also reduces winding copper.

Q.10 The winding MMF in rotating machines depends on
(a) winding arrangement
(b) winding current
(c) air gap length, slot openings, etc.
(d) both winding arrangement and winding current

Q.11 What will be the produced mmf (in Amp-turns) in a coil, if the coil has 160 turns and carries a current of 0.15 A?
(a) 32
(b) 24
(c) 16
(d) 8

Q.12 A chording angle ($\in$) is defined as the angle by which coil span departs from

(a) 90° electrical space degrees
(b) 180° electrical space degrees
(c) 360° electrical space degrees
(d) Any of the above

Q.13 The emf induced in a conductor of machine driven at 600 rpm, the peak value of flux density is 1.0 Wb/m^2, diameter of machine 2.0 meter and length of machine 0.30 m is
(a) 41.83 V (b) 29.58 V
(c) 9.42 V (d) 18.84 V

Q.14 The developed electromagnetic force and/or torque in electromechanical energy conversion systems, acts in a direction that tends to
(a) decrease the stored energy at constant mmf
(b) increase the stored energy at constant mmf
(c) decrease the co-energy at constant mmf
(d) increase the stored energy at constant flux

Q.15 If ϕ_m is the maximum value of flux due to any one phase, the resultant flux in 3-phase and 2-phase ac machines would be given by
(a) $2\,\phi_m$ and $1.5\,\phi_m$; both rotating
(b) $1.5\,\phi_m$, rotating and ϕ_m, standstill
(c) $1.5\,\phi_m$, and ϕ_m; both standstill
(d) $1.5\,\phi_m$, and ϕ_m; both rotating

Q.16 In non-salient pole synchronous machine the distribution of field mmf around air gap is a
(a) sinusoidal wave
(b) rectangular wave
(c) stepped triangular wave
(d) flat topped stepped wave

Q.17 If the dimensions of all the parts of a synchronous generator, and the number of field and armature turns are doubled, then the generated voltage will change by a factor of
(a) 2 (b) 8
(c) 4 (d) 1

Q.18 For eliminating nth harmonic from the e.m.f generated in the phase of a 3-ϕ alternator, the chording angle.
(a) $n \times$ full pitch (b) $\frac{2}{n} \times$ full pitch
(c) $\frac{1}{n} \times$ full pitch (d) $\frac{3}{n} \times$ full-pitch

ANSWER KEY // STUDENT ASSIGNMENT

1. (b)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (a)
6. (c)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (d)
11. (a)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (d)
16. (d)	**17.** (b)	**18.** (c)		

HINTS & SOLUTIONS // STUDENT ASSIGNMENT

1. (b)

We know that inductance,

$$L \propto \mu\ \text{(Permeability)} \propto \mu_0 \mu_r \propto \mu_r$$

Since relative permeability of iron is more than that of air ($\mu_r = 1$) therefore, with iron-cored coil L will be more while with air-cored coil L will be less.

2. (a)

In order to eliminate nth harmonic, short pitching angle = θ_{sp}.

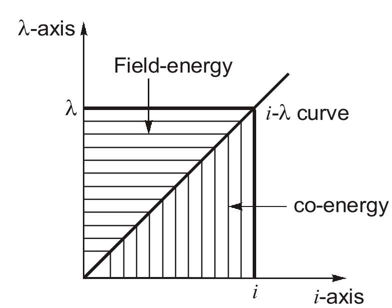
$$\frac{n\theta_{sp}}{2} = 90°$$

Here, $n = 5$

$$5 \times \frac{\theta_{sp}}{2} = 90°$$

$\Rightarrow$ $\theta_{sp} = 36°$

3. (a)



Where, $\lambda = N\phi$ = Flux linkage
Field energy is the energy absorbed by the magnetic system to establish flux ϕ.
For a linear electromagnetic circuit

$$\text{Field energy} = \text{Co-energy} = \frac{1}{2}\lambda i$$